Electricity and Magnetism, Exam 4, 28/04-01/05 2017

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12 questions; with answers

This is a multiple-choice exam. Write your name and student number on the answer sheet. Clearly mark the answer of your choice on the answer sheet. Only a single answer is correct for every question. The score will be corrected for guessing. Use of a (graphing) calculator is allowed. You may make use of the formula sheet (provided separately). The same notation is used as in the book, i.e. a bold-face **A** is a vector, T is a scalar.

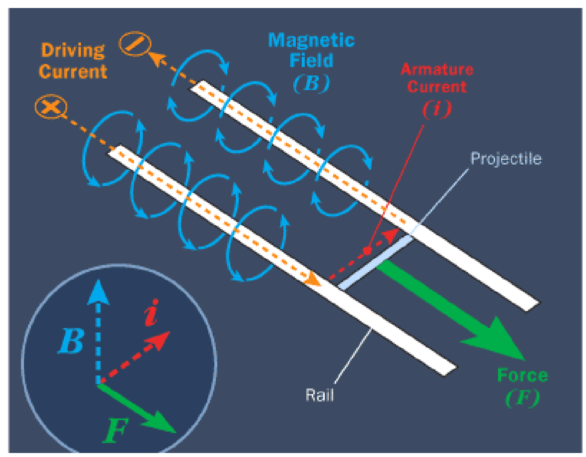
Table of correct answers

Question	**Correct answer**
1	**B**
2	**B**
3	**A**
4	**A**
5	**D**
6	**C**
7	**D**
8	**A**
9	**A**
10	**D**
11	**B**
12	**C**

1. A pair of parallel conductors (rails) are firmly mounted on a horizontal surface. A conducting nonmagnetic bar, orthogonal to the rails, can freely slide along (but not across) the rails maintaining an electrical contact to both of them. At time *t=0* the rails are connected to the power supply as shown in the figure. To which direction will the bar move?

A. To the left
B. To the right V
C. Depends on the exact material of the bar
D. The bar will not move

Answer B: The two rails act like wires, with a magnetic field circulating around each rail. The force lines of the magnetic field run in a counterclockwise circle around the positive rail and in a clockwise circle around the negative rail. The net magnetic field between the rails is directed vertically.
The Lorentz force is directed perpendicularly to the magnetic field and to the direction of the current (=charges) flowing across the bar.

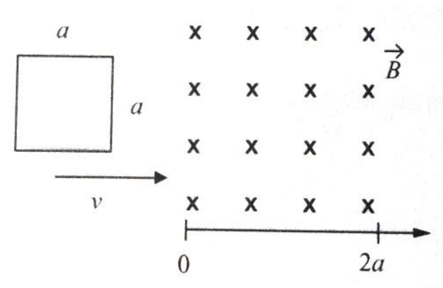


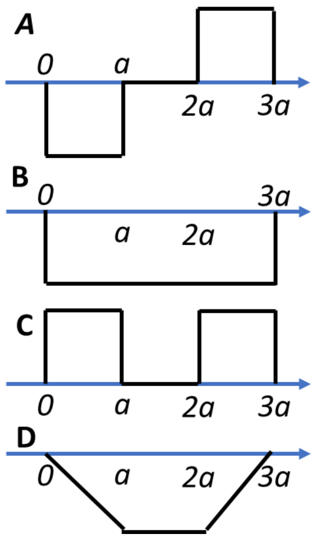
Check up the youtube for the "railgun" and have fun!

2. In the previous question, the force exerted upon the bar
A. Is proportional to the current
B. Is proportional to the square of the current V
C. Depends on the exact material of the bar
D. There is no force

Answer B: The net magnetic field is proportional to the current while the Lorentz force is proportional to a product of the magnetic field and the current (=charges).
NB: Answer B is more correct than "Depends on the exact material of the bar" as it provides the right functional dependence.

3. A square loop of side *a* moves with the constant speed *V* into a region in which a magnetic field of magnitude *B* exists perpendicular to the plane of the loop as shown in the figure. The emf induced in the loop as it enters, moves through, and exits the region of the magnetic field, is shown in the graphs below. Which graph is correct?





Answer A: The induced ems is proportional to minus *change* in the magnetic flux. As the loop enters the region, the magnetic flux increases so that a constant emf (*V* is constant) is induced. As the loop moves fully into the magnetic field region, there is no change of flux any longer so that emf=0. Upon exiting the magnetic field region, the situation is reversed with emf changing the sign.

4. A long solenoid with radius *a* and *n* turns per unit length carries a time-dependent current *I(t)* in the $\hat{\boldsymbol{\phi}}$ direction. Find the electric field at a distance *s* from the axis inside the solenoid in quasistatic approximation.

Tip: The magnetic field inside the solenoid in the quasistatic approximation is $\mathbf{B} = \mu_0 n I\,\hat{\mathbf{z}}$

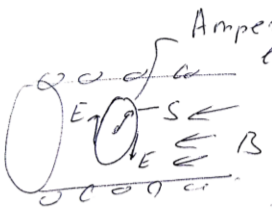
A. $\mathbf{E} = -\frac{\mu_0 n s}{2}\frac{dI(t)}{dt}\hat{\boldsymbol{\phi}}$ V

B. $\mathbf{E} = -\frac{\mu_0 n s}{2} I(t)\hat{\boldsymbol{\phi}}$

C. $\mathbf{E} = -\mu_0 n s\frac{dI(t)}{dt}\hat{\boldsymbol{\phi}}$

D. $\mathbf{E} = -\frac{\mu_0 n s}{2} I(t)\hat{\mathbf{z}}$

Answer A:

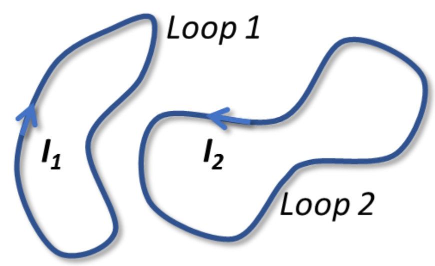


$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi}{dt}$$

$$E \cdot 2\pi s = -\frac{d}{dt} \mu_0 n \pi s^2 I(t)$$

$$\vec{E} = -\frac{\mu_0 n s}{2} \frac{dI}{dt} \hat{\varphi}$$

5. The mutual inductance M_{12} of the two loops depicted in the figure and currying the currents I_1 and I_2, depends on:



A. Current I_1

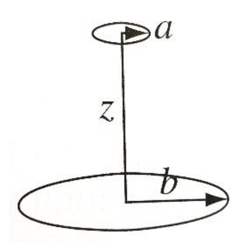
B. Current I_2

C. Currents I_1 and I_2

D. The sizes, shapes and relative positions of the loops V

Answer D: The mutual inductance is purely geometrical quantity that depends on the sizes, shapes and relative positions of the loops.

6. A small loop of wire (radius a) is held at a distance z above the center of a large loop (radius b), as shown in the figure. The planes of the two loops are parallel, and perpendicular to the common axis. The little loop is so small that you may consider the field of the big loop $\mathbf{B} = \frac{\mu_0 I_a}{2} \frac{b^2}{(b^2+z^2)^{\frac{3}{2}}} \hat{\mathbf{z}}$ to be essentially constant. The little loop is so small that you may treat it as a magnetic dipole with the field $\mathbf{B} = \frac{\mu_0 I_b a^2}{4} \left(2cos\theta\, \hat{\mathbf{r}} + sin\theta\, \hat{\boldsymbol{\theta}}\right)$. Find the mutual inductance M of the loops. (Tip: recall properties of the mutual inductance before running calculations!)



A. $M = \frac{\mu_0 \pi I_a I_b}{2} \frac{a^2 b^2}{(b^2+z^2)^{\frac{3}{2}}}$

B. $M = \frac{\mu_0 \pi I_a I_b}{2} \frac{a^2 b^2}{(b^2+z^2)^{\frac{1}{2}}}$

C. $M = \frac{\mu_0 \pi}{2} \frac{a^2 b^2}{(b^2+z^2)^{\frac{3}{2}}}$ V

D. $M = \frac{\mu_0 \pi}{2} \frac{a^2 b^2}{(b^2+z^2)^{\frac{1}{2}}}$

Answer C: As the mutual inductance does not depend on through which loop to run the "test" current, we should choose the simplest situation for our calculations. Because of simplicity of the magnetic field produced by the large loop, we use its magnetic field.

$$\Phi_a = M \cdot I_B = B_B \cdot \pi a^2$$

$$M = \pi a^2 \frac{B_B}{I_B} = \frac{\mu_0 \pi}{2} \frac{a^2 b^2}{(b^2+z^2)^{3/2}}$$

You can also reject A and B as current-dependent (the inductance is not).

7. Find the emf induced in a square loop with sides of length *a* lying in the *yz*-plane in a region in which the magnetic field changes over time as $\mathbf{B}(t) = B_0 e^{-t/t_0}\hat{\boldsymbol{\imath}}$.

A. $emf = 0$

B. $emf = B_0 e^{-t/t_0}$

C. $emf = \frac{a^2 B_0}{t_0}$

D. $emf = \frac{a^2 B_0}{t_0} e^{-t/t_0}$ V

Answer D: As the field is orthogonal to the loop,

$$\mathcal{E} = -\frac{d\Phi}{dt} = \frac{d}{dt}\left(a^2 B_0 e^{-t/t_0}\right)$$

$$= -\frac{a^2 B_0}{t_0} e^{-t/t_0}$$

8. The magnetic field in a certain region is given by the expression $\mathbf{B}(t) = B_0 cos(kz - \omega t)\hat{\boldsymbol{\jmath}}$. Find the curl of the induced electric field at that location.

A. $\nabla \times \mathbf{E} = -\omega B_0 sin(kz - \omega t)\hat{\boldsymbol{\jmath}}$ V

B. $\nabla \times \mathbf{E} = -\omega B_0 cos(kz - \omega t)\hat{\boldsymbol{\jmath}}$

C. $\nabla \times \mathbf{E} = -\left(\omega + k\frac{\partial z}{\partial t}\right)\omega B_0 sin(kz - \omega t)\hat{\boldsymbol{\jmath}}$

D. $\nabla \cdot \mathbf{E} = -\omega B_0 sin(kz - \omega t)\hat{\boldsymbol{\jmath}}$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\frac{\partial}{\partial t} B_0 \cos(kz-\omega t)\cdot\vec{j}$$
$$= \cancel{-}\,\omega B_0 \sin(kz-\omega t)\cdot\vec{j}$$

Answer A.

9. As the current in an induction coil changes uniformly from 1 A to 6 A in 0.1 s, emf of -50 V is induced. What is inductance of the coil?

A. 1 H *V*

B. 0.5 H

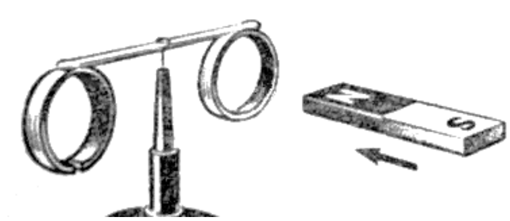
C. 1 T

D. 4 H

$$\mathcal{E} = -L\frac{dI}{dt}; \quad L = -\frac{\mathcal{E}}{dI/dt}$$
$$L = -\frac{50\,V}{(6-1)A/0.1\,s} = 1H$$

Answer A: Inductance has units of Henries (H)!

10. The magnet is pushed inside the metal *nonmagnetic* ring as shown in figure. What is the direction of the movement of the ring?



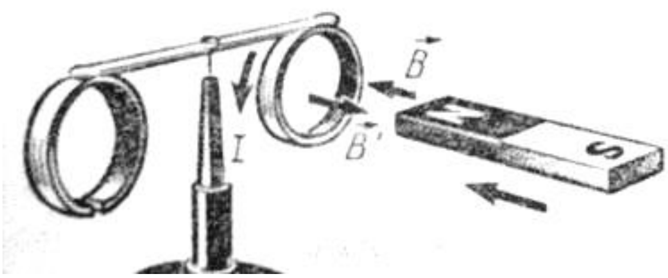
A. The ring does not move because it is nonmagnetic

B. It depends on the speed of the magnet

C. Toward the magnet

D. Opposite the magnet *V*

Answer D: As the magnet approaches, the magnetic flux through the ring increases. Therefore, the current will be induced to produce the ring magnetic field to counteract the flux increase.



As a result, the ring will be repelled from the magnet.

11. Airbus 380 has a wingspan of 80 m and travels horizontally with a speed of 1080 km/h. Find emf induced between the tips of the wings. The vertical component of the Earth's magnetic field amounts to $5 \cdot 10^{-5}$ T.

A. 0.6 A

B. 1.2 V V

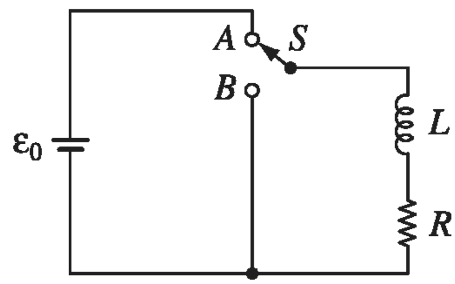
C. 0.6 V

D. 0

Answer B:

$$\mathcal{E} = Bhv = 5 \cdot 10^{-5} \cdot 80 \cdot \frac{1080}{3.6} = 1.2V$$

12. The circuit has been connected for a long time when suddenly, at time t = 0, switch S is thrown from A to B, bypassing the battery. What is the current *I(t)* at any subsequent time *t*?



A. $I(t) = 0$ because the battery is disconnected

B. $I(t) = \frac{\varepsilon_0}{R}$

C. $I(t) = \frac{\varepsilon_0}{R} e^{-Rt/L}$ V

D. $I(t) = \frac{\varepsilon_0}{R}\left(1 - e^{-Rt/L}\right)$

$$-L\frac{dI}{dt} = IR$$

$$\frac{dI}{dt} + \frac{R}{L} I = 0 \Rightarrow I = A \cdot e^{-R/L \cdot t}$$

at $t = 0$, $I = \frac{\varepsilon_0}{R}$, i.e. $A = \frac{\varepsilon_0}{R}$

Answer C. Another (smart) way of solving the problem without any math is to recall that the energy must be stored in the magnetic field of the inductance. Therefore, this energy should go somewhere (= to the resistor) which must take some time. So the right solution should (i) produce some decrease in time, and (ii) decay to zero asymptotically. Solutions A and B give a constant (wrong), solution D describes an increasing current (was treated at the lecture).